PUBLICATIONS OF THE SCOTTISH COUNCIL
FOR RESEARCH IN EDUCATION

XXVIII

ADDITION AND SUBTRACTION FACTS AND PROCESSES

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PREFACE

The first project of the Early Number Teaching Panel under the convenership of Mr Jas. Younie was to investigate the child's knowledge of number on entering school. Its report on this topic was published in 1942 under the title of *The Early Development of Number Concepts*. The step that naturally followed was to assess the child's knowledge of number on his transition from the Infant Division to the Primary Division of the school with a view to formulating a course of study in arithmetic. By following up the pupils tested into the Primary School it was also sought to learn whether the efficiency attained by the constant training on the number combinations in the Infant Division was maintained in the Primary School. The information obtained to decide these issues is presented in this report.

The form in which the tables now appear in this monograph, the technique whereby the relative difficulty of the number combinations can be expressed and the difficulty of the carrying and borrowing operations assessed, the Panel owes to Mr Mowatt. The errors made in the tests are being studied by another member of the Panel.

The Panel desires to express its appreciation of the willingness of the staffs of the schools approached to apply the tests.

W. L. McKINLAY
Convener of Panel on
Early Number Teaching

COMMITTEE ON PRIMARY SCHOOL SUBJECTS

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CONTENTS

PAGE

PREFACE 5

COMMITTEE 6

INTRODUCTION 9

ADDITION AND SUBTRACTION FACTS . . . 13
  Addition Facts (with Appendix) 16
  Subtraction Facts (with Appendix) . . . 31

ADDITION AND SUBTRACTION PROCESSES . 43
  Addition Processes 46
  Subtraction Processes 54

RECOMMENDATIONS 66

INTRODUCTION

Aim of Investigation

The Early Number Teaching Panel of the Committee on Primary School Subjects investigated the number knowledge of children entering school; the results of this study are published in *The Early Development of Number Concepts*. The Panel then decided to inquire into the knowledge of number which the pupil normally acquires during his school life in the Infant Division. Its aims were to secure evidence on which a satisfactory syllabus in arithmetic for the Infant Division could be based; to determine the degree of mastery attained by pupils at the point of transfer from the Infant Division to the Primary Division of the Scottish school system; to examine the contention that the later work in arithmetic is handicapped by inadequate command by the pupils in the Primary Division of the fundamental facts and processes; and by repeating the tests on the same pupils at six-monthly intervals to ascertain if there is any loss owing to discontinuance of practice.

Scope of Investigation

The Panel has sought in the first place precise information regarding the pupils' mastery of the Addition and Subtraction Facts and of the various steps in the Addition and Subtraction Processes. As it is extremely difficult to frame examples of the Processes which comprise all the Facts, the Panel decided to have the Facts tested separately from the Processes.

SUBJECTS

As it was impossible to secure a thoroughly representative sample of pupils as in the Scottish individual intelligence survey,[1] it was decided to test selected schools in different counties which might serve as a cross-section of Scottish schools, namely the Cities of Edinburgh and Glasgow, and the Counties of Aberdeenshire, Ayrshire, Bute, Clackmannan, Fife and Selkirk.

Instead of testing an age-group, it was decided that for the purpose the Panel had in view all the pupils in the last six months of the Infant Division should take the tests, and that these pupils should be followed up so far as possible and retested from time to time.

As it was essential that each pupil should perform every operation, no time limit could be imposed. Nevertheless, although the Panel's investigation was planned primarily to measure accuracy, the opportunity was taken to secure measures of rate of work by instructing the teachers to record on pupils' work-sheets the time taken by each pupil for the hundred combinations in intervals of one minute. This information, it was thought, might be of value in discerning the relationship between accuracy of answers and promptness, and by taking 'soundings' at different stages of the pupils' careers in ascertaining whether rate and accuracy increased *pari passu* or whether one improves at the expense of the other.

Scripts were returned from 47 schools in 8 areas in Scotland for Addition and Subtraction Facts and Addition and Subtraction Processes, the children tested being all in the last six months of the Infant Division. Sets of Addition and Subtraction completed papers were received from 1,427 children, 714 boys and 713 girls, of which 1,315 were used, 665 boys and 650 girls. The total population of children in Scotland between 6½ and 7 was 41,000. The rejected papers numbered 106; 48 boys and 58 girls had not completed more than 90 facts in the Addition or Subtraction paper. Inspection of these papers showed that incompleteness was in many accidental and not necessarily due to low level of performance (some of those who were soonest finished and had most correct appeared to have jumped a line merely through speed or eagerness, others had performed the wrong process), but the time recorded for 98 out of the 212 papers (Addition and Subtraction) was over 30 minutes. The remaining 6 could not be included because of failure to use the regular test form, etc.

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[1] *Cf.* A. M. Macmeeken, *The Intelligence of a Representative Group of Scottish Children.* Publication of The Scottish Council for Research in Education, No. XV. London: University of London Press, Ltd., 1939.

The distribution of children tested in June was as follows:

Area	Numbers given on nominal roll			Numbers of papers used		
	Boys	Girls	Total	Boys	Girls	Total
Aberdeenshire .	50	34	84	47	32	79
Ayrshire .	100	113	213	97	105	202
Bute . .	14	16	30	14	16	30
Clackmannan .	48	45	93	39	40	79
Edinburgh .	99	115	214	91	104	195
Fife . .	165	170	335	149	144	293
Glasgow .	194	176	370	189	168	357
Selkirk . .	44	44	88	39	41	80
Totals. .	714	713	1,427	665	650	1,315

The same pupils were retested in December 1945 and in June 1946, with the exception of those who had left school and those who were absent on the days of the later tests.

ADDITION AND SUBTRACTION FACTS

Type of Investigation

Investigations into number combinations have been classified into 'learning' studies and 'status' studies, the latter being those in which measures of knowledge are secured at points in time distant, it may be a year or more, from the conclusion of instruction in the facts.[1] The studies of the Panel on Early Number Teaching are 'status' studies. An investigation into the types of errors is being undertaken by a member of the Panel.

The order of difficulty of the combinations may vary in the two types of studies, 'learning' and 'status'; a difficulty may occur at one place in the initial stages of learning and at quite a different place at later stages.[2] It must consequently not be assumed that the order disclosed by the Panel's inquiry indicates the difficulty of learning the combinations; it merely reveals the success which the pupil has attained in writing correct answers to the combinations set in a formal fashion at the stage of his school progress when the test was applied. The efficiency displayed is naturally conditioned by the teaching method adopted, the learning procedure followed by the pupil,[3] the time devoted to arithmetic, etc.

---

[1] William A. Brownell and Doris V. Carper, *Learning the Multiplication Combinations.* Durham, N.C.: Duke University Press, 1943. P. 26.

[2] P. 32.

[3] Brownell and Carper have shown (p. 16) that children learn by processes quite unlike those by which they are taught. They may be taught the combinations by repetition or drill methods but do not learn them through repetition; instead they inject into the process some meaningful relationship or scheme of organisation.

It also depends on the emphasis laid on each of the combinations in teaching. Thus inability to handle the zero facts may not be due to their inherent difficulty but to neglect of these during the early stage of instruction. As Brownell and Carper maintain for the multiplication facts: 'there is no *final* order of difficulty, apart from the learner and all the conditions which govern his behaviour.'[1] In fact the value for instruction which the studies into the relative difficulty of the number combinations possess is mainly negative or diagnostic; they serve to locate deficiencies. They should be complemented by studies of the thought processes employed by pupils to discover the way in which the manipulations are actually performed.

The criteria of mastery may either be accuracy, rate of performance, or type of thought process employed. In the Scottish investigation the measure of learning was accuracy.

PROCEDURE

For testing purposes the Addition and the Subtraction Facts were arranged in strictly random fashion; randomisation was effected by following the table of random numbers in *Statistical Tables for Biological, Agricultural and Medical Research*, by R. A. Fisher and F. Yates, reproduced in the Appendix (Table 18) in E. F. Lindquist's *Statistical Analysis in Educational Research*. The selected combinations, a hundred in number, arranged themselves naturally into ten rows of ten each. To eliminate or compensate for the effects of adaptation or fatigue the rows were rotated[1] by printing line (*a*) first on one sheet, second on the next, and so on. By distributing to pupils the sheets so rotated copying is also avoided, as pupils sitting next to one another begin at different points. As addition may be performed either upwards or downwards and as the combinations were arranged for upward addition, only schools teaching upward addition were used for testing purposes. The equation form was rejected for much the same reason as the adding upwards and downwards confusion, and the Panel's unanimous decision is that this arrangement, doubtless introduced by publishers to save space, should be vetoed in Infant Division teaching. As some studies of the fundamental process in arithmetic are vitiated by pupils performing the wrong operation—multiplication or subtraction for addition—one type of operation only was allocated to one day and each example had prefixed to it its appropriate sign.[2] That the task might not appear too formidable five rows were printed on one side of the sheet and five on the other.

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[1] P. 36; for example the present investigation is restricted to *written* number facts. A pupil might give the correct answer orally to 9+7, but write the answer 61.

---

[1] For the value of the method of rotation see W. A. McCall, *How to Experiment in Education*. New York: The Macmillan Company, 1926. P. 33.

[2] For typical sheets see Appendices I and II, pp. 30 and 41.

ADDITION FACTS

The results obtained in the Addition Facts are presented in percentage form in Table I (*a*), (*b*) and (*c*).

All the results are high, thus causing small variations in the percentages correct. Had they been otherwise arranged the results might well lead to the idea that they are random values, but an examination of Table I disproves this. Table I makes clear the following points:

1. The results for the facts which contain zero, *i.e.* of type $0+x$ and $x+0$, are abnormal and hence must be considered separately.
2. If the facts containing zero are excluded a definite relationship is revealed. With a few exceptions, as the first digit in the fact increases from 1 to 9 the degree of difficulty increases, and as the second digit increases from 1 to 9 the degree of difficulty increases. This relationship is really remarkable when one considers the very small variations in the values. It is shown even more convincingly in Table II.
3. The doubles, *i.e.* facts with two equal digits, are printed in heavier type in Table I. They are definitely less difficult than might have been expected statistically.
4. A comparison of the results for the tests given in Table I shows that for all the facts, except those containing zero, there is a marked improvement throughout. This is corroborative evidence of what seems to be obvious, namely, the percentages correct increase with the mental growth of the group.

With such small differences in the percentages correct, the relationship between these and between the first and second digits makes it certain that they are not random values (see Table II). It seems certain, therefore, that at each stage in the school course each fact has its own distinct value.

In the Infant Division the Addition Facts will each involve its own mental process, but the operation of such mental processes may be disturbed by other factors, *e.g.* fatigue, noise. These extraneous factors will give rise to random variations in the results. The actual percentage number correct in any fact will therefore be determined by

(i) the inherent difficulty of the mental operation involved in the addition of the two numbers;

(ii) the chance extraneous conditions and events at the time the actual mental process was in operation.

Over the large group of pupils these random factors will nevertheless operate on all facts equally and will have the effect of decreasing the percentage number correct. Actually this decrease must be very small. The percentage number correct in any fact is not an absolute but a relative measure of its difficulty.

It would appear, but it remains to be proved by further follow-up tests, that a normal group of pupils will ultimately reach a stage where addition facts in themselves will present no difficulty and any errors in them will be due to chance only. If and when such a stage is reached, the results will be random results, and there will be no relationship between the facts and percentages correct. Thus, if the test in Addition Facts is given to a normal group of pupils at the Qualifying stage it is almost certain that there would only be chance variations in the results.

The determination of this ultimate stage for a normal group would appear to be the logical conclusion to the present investigation, as it will be of vital importance in the preparation of schemes of work in arithmetic for the Primary school.

While the relationship between fact and percentages correct, referred to above, is very marked, there are a few discrepancies.

FACTS CONTAINING ZERO

It has been pointed out that the facts containing zero require separate consideration. The following results will give some idea as to the position:

Average result for	June	Dec.	Difference
All facts . . .	94·55	95·28	+0·73
Facts not containing zero	94·37	95·87	+1·50
$\begin{array}{r}x\\ +0\\ \hline\end{array}$ facts . . .	97·53	96·08	−1·45
$\begin{array}{r}0\\ +x\\ \hline\end{array}$ facts . . .	93·28	90·72	−2·56

These results show that

(*a*) the $\begin{array}{r}0\\ +x\\ \hline\end{array}$ facts are definitely more difficult than the $\begin{array}{r}x\\ +0\\ \hline\end{array}$ facts.

(*b*) There is a marked deterioration in both $\begin{array}{r}0\\ +x\\ \hline\end{array}$ and $\begin{array}{r}x\\ +0\\ \hline\end{array}$ facts from June to December. It will be remembered that in all other facts there is an improvement. This may be due to the pupils having started the multiplication facts between June and December, but the anomaly requires further investigation. The function of the digit 0 both in addition and in multiplication is unique and apparently gives rise to a type of difficulty not associated with the other digits.

RANKING ORDER

A further conclusive proof that the Addition Facts are essentially separate entities, each developing as the group develops, is obtained by considering the ranking order of the facts as determined by their percentages correct (see Table III). There is available the ranking orders of decreasing difficulty as given by

(i) June 1945 test.
(ii) December 1945 test.
(iii) June 1946 test.

Here again the facts containing zero have a disturbing effect, and in Table III the orders have been determined without including the facts of type $x+0$ and $0+x$. Referring to this Table, it is found that for many of the facts the ranking orders are very close indeed. The high values for the rank correlations between test and test are definitely significant, and prove conclusively that each of the facts, excluding those containing zero, has its own particular degree of difficulty.

Another ranking is given in Murray's investigation.[1] Murray's pupils were, however, completing their second session of the Primary Division, that is, their average age was nine years.[2] They had also probably lost their familiarity with number combinations presented in isolation.

---

[1] *Studies in Arithmetic*, Vol. I. Publication No. XIII, Scottish Council for Research in Education. London: University of London Press, 1939. Pp. 79–115.

[2] *Ibid.*, p. 82.

TABLE I

Showing the percentage number who were successful in each of the

ADDITION FACTS

in June 1945, December 1945, June 1946

The digits in the left-hand column indicate the first number of the equation form or the upper number of the combination; the digits along the top row the second number of the equation form or the lower number of the combination—thus 4 + 3, *i.e.* $\begin{array}{r}4\\ \underline{+3}\end{array}$ = 96 per cent. correct in June 1945.*

June 1945 (*a*)

	0	1	2	3	4	5	6	7	8	9
0	**98·5**	91·6	93·7	92·1	93·6	94·2	94·1	93·7	92·7	93·8
1	96·5	**97·3**	97·1	97·6	97·7	97·2	97·3	97·3	97·3	95·1
2	98·0	97·1	**98·0**	96·6	95·8	96·0	96·0	95·6	96·3	95·6
3	98·2	96·7	96·0	**97·3**	95·8	95·4	94·1	95·0	94·7	93·0
4	97·6	96·3	95·3	96·0	**97·1**	96·7	94·2	95·1	93·8	92·0
5	97·5	95·7	95·8	94·9	95·3	**97·4**	94·0	92·8	90·3	91·0
6	97·6	95·7	94·2	94·0	93·6	94·3	**95·4**	94·0	90·3	92·3
7	97·7	96·3	95·8	94·4	93·6	91·4	91·2	**95·4**	91·1	90·0
8	98·2	96·3	94·6	94·0	92·3	92·2	90·5	90·0	**94·6**	90·3
9	96·5	95·3	94·6	92·0	91·4	89·8	90·0	88·2	89·0	**93·4**

N = 1,300
Mean of all scores = 94·55
Standard Deviation of scores = 12·31

---

* The interpretation of the equation form presents difficulty. The subtraction pattern is taken as the model. Thus $4 - 3 = \begin{array}{r}4\\ \underline{-3}\end{array}$ Accordingly $4 + 3 = \begin{array}{r}4\\ \underline{+3}\end{array}$

(*Cf.* J. Morrison, *The Teaching of Arithmetic.* Publication of The Scottish Council for Research in Education, No. XXI. London: University of London Press, Ltd., 1944. P. 18.)

December 1945 (*b*)

	0	1	2	3	4	5	6	7	8	9
0	**99·1**	89·4	90·5	91·0	91·2	90·8	91·3	90·7	90·7	90·9
1	94·6	**98·5**	97·9	97·9	98·4	98·7	98·5	98·3	98·6	96·9
2	96·2	97·6	**97·6**	97·6	97·3	97·2	97·1	97·1	96·4	96·9
3	96·6	98·2	97·2	**98·3**	96·5	96·7	96·1	96·4	96·2	95·0
4	96·6	98·1	97·3	96·0	**98·5**	97·6	95·6	95·8	94·9	94·1
5	95·5	96·9	96·9	95·9	96·7	**97·7**	95·1	93·6	92·4	94·9
6	95·5	97·4	97·0	95·7	95·3	95·1	**96·8**	93·3	92·9	93·6
7	96·5	97·9	96·4	95·1	94·0	93·5	93·0	**97·1**	91·3	92·1
8	96·3	98·3	96·6	94·6	93·8	93·7	92·7	92·0	**96·7**	92·8
9	95·8	96·9	95·7	94·3	93·1	94·2	92·3	91·6	93·1	**96·4**

N =1,113
Mean of all scores = 95·28
Standard Deviation of scores = 12·28

June 1946 (*c*)

	0	1	2	3	4	5	6	7	8	9
0	**99·2**	91·1	91·7	91·2	92·0	92·5	92·2	90·7	91·3	91·9
1	97·0	**98·1**	98·1	98·5	98·3	98·6	98·3	97·9	98·1	97·8
2	97·4	98·7	**99·0**	97·8	97·7	98·0	97·4	97·2	97·3	97·2
3	97·5	98·8	97·8	**98·2**	97·7	97·5	97·5	97·4	97·0	96·8
4	97·7	97·9	97·1	97·1	**97·9**	97·8	96·4	96·0	97·3	94·1
5	97·6	97·9	98·1	97·7	96·9	**97·7**	97·3	96·3	94·9	96·6
6	97·5	97·8	96·8	97·7	97·2	97·0	**97·5**	95·5	94·3	95·2
7	97·9	97·4	97·4	97·0	95·6	95·4	95·3	**98·1**	94·7	93·5
8	97·7	98·5	97·8	96·0	97·2	94·7	96·1	94·1	**97·4**	94·6
9	97·6	98·3	96·9	95·4	95·0	95·1	94·5	92·7	93·0	**97·4**

N =1,036
Mean of all scores = 96·48
Standard Deviation of scores = 9·15

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TABLE II

Showing the percentage number successful in Addition Combinations in which a given digit is (*a*) the first number of the equation or the upper number of the combination; and (*b*) the second number of the equation or the lower number of the combination, for June 1945, December 1945 and June 1946

Zeros are omitted

June 1945

Combination in which the indicated digit is the	1	2	3	4	5	6	7	8	9
(*a*) first number of the equation form or upper number of the combination . .	97·1	96·3	95·3	95·2	94·1	93·8	93·2	92·8	91·5
(*b*) second number of the equation form or lower number of the combination . .	96·3	95·7	95·2	94·7	94·5	93·6	93·7	93·0	92·5

December 1945

Combinations in which the indicated digit is the	1	2	3	4	5	6	7	8	9
(*a*) first number of the equation form or upper number of the combination . .	98·2	97·2	96·8	96·5	95·6	95·2	94·5	94·6	94·2
(*b*) second number of the equation form or lower number of the combination . .	97·8	96·9	96·2	96·0	96·0	95·2	95·0	94·7	94·7

June 1946

Combination in which the indicated digit is the	1	2	3	4	5	6	7	8	9
(*a*) first number of the equation form or upper number of the combination . .	98·2	97·7	97·3	97·1	96·9	96·7	96·1	96·0	95·9
(*b*) second number of the equation form or lower number of the combination . .	98·2	97·8	97·6	96·8	97·0	96·6	96·0	96·3	95·4

TABLE III (*a*)

Showing the ranking order in decreasing difficulty of the

ADDITION FACTS

according to June 1945, December 1945 and June 1946, also average of three tests

All facts containing zeros have been omitted

The digits in the left-hand column indicate the first number of the equation form or the upper number of the combination; the digits along the top row the second number of the equation form or the lower number of the combination—thus 4+3, *i.e.* 4 June 1945, 58th; December 1945, 36th; June 1946, 34th.
+3

	+1	+2	+3	+4	+5	+6	+7	+8	+9
1	75 79 68 94	70 70 67 89	79 68 77 97	80 76 75 99	72 81 78 98	73 78 74 96	76 74 62 86	77 80 69 95	43 52 59 71
2	71 63 79 91	81 64 81 100	66 66 58 84	57 61 52 79	59 59 66 81	61 57 43 77	50 56 37 70	64 40 39 69	51 51 35 68
3	68 72 80 93	60 58 57 80	74 73 72 92	56 43 51 76	47 47 48 67	31 37 50 56	41 43 47 60	39 38 32 55	22 26 26 44
4	63 71 64 85	45 60 33 66	58 36 34 63	69 77 65 90	67 65 61 83	32 31 24 49	42 34 21 51	26 25 40 46	16 20 5 34

Correlations:

June 1945 and December 1945 =·945
December 1945 and June 1946 =·894

	+1	+2	+3	+4	+5	+6	+7	+8	+9
5	52	55	40	44	78	30	21	7	11
	50	53	35	45	67	30	16	6	24
	63	70	55	29	54	41	23	11	25
	75	78	58	62	87	48	40	28	38
6	53	33	28	25	34	48	27	9	19
	62	54	33	29	27	48	13	9	15
	56	27	53	38	30	49	18	6	14
	74	57	54	45	47	65	39	27	36
7	62	54	35	24	14	13	49	12	7
	69	41	28	19	14	10	55	1	4
	46	45	31	19	17	15	71	9	3
	82	64	50	42	33	23	73	25	22
8	65	38	29	20	18	10	4	37	7
	75	44	23	18	17	7	3	46	8
	76	60	20	36	10	22	4	44	8
	88	61	43	41	35	31	23	59	26
9	46	36	17	15	3	6	1	2	23
	49	32	22	12	21	5	2	11	39
	73	28	16	12	13	7	1	2	42
	72	53	37	30	29	24	20	21	52

TABLE III (*b*)

Showing the ranking order in decreasing difficulty of the

ADDITION FACTS

according to June 1945, December 1945 and June 1946, also average of three tests

All facts containing zeros have been included

The digits in the left-hand column indicate the first number of the equation form or the upper number of the combination; the digits along the top row the second number of the equation form or the lower number of the combination.

	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
0	100	16	30	19	27	40	38	30	23	32
	100	1	2	7	8	5	9	3	3	6
	100	2	7	3	6	9	7	1	4	5
	100	1	8	3	12	18	17	6	4	11
1	75	84	80	91	94	83	84	84	84	51
	32	95	86	86	94	99	95	91	98	67
	39	85	85	94	91	96	91	79	85	73
	48	93	88	96	98	97	95	85	94	63
2	97	80	96	77	63	68	68	59	71	59
	50	81	81	81	78	76	73	73	53	67
	52	97	99	73	66	84	52	45	49	45
	77	90	99	81	72	74	70	62	61	59
3	98	78	68	84	63	56	38	50	49	25
	59	90	76	91	57	63	49	53	50	36
	59	98	73	90	66	59	59	52	39	35
	84	92	73	91	69	58	46	51	45	34

	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
4	91	71	53	67	80	78	40	51	32	17
	59	89	78	48	95	81	42	45	34	29
	66	81	43	43	81	73	33	29	49	13
	80	83	57	54	89	79	39	41	36	24
5	90	61	63	49	53	89	34	24	7	11
	59	67	67	47	63	85	39	24	15	34
	64	79	85	66	37	66	49	32	20	34
	76	68	71	49	53	86	38	30	16	28
6	91	61	40	34	27	43	56	34	7	21
	41	80	72	43	39	37	66	22	18	24
	59	73	35	66	45	39	59	27	15	23
	67	66	47	44	35	37	56	29	15	26
7	94	71	63	44	27	14	13	56	12	5
	57	86	53	37	28	23	19	73	9	13
	81	52	52	39	28	25	24	85	18	12
	78	75	55	40	32	23	22	65	13	7
8	98	71	45	34	21	20	10	4	45	7
	52	91	59	32	27	26	16	12	63	17
	66	94	73	29	45	18	31	13	52	17
	82	87	52	33	31	25	21	9	50	14
9	75	53	45	17	14	3	5	1	2	26
	45	67	43	31	20	30	14	11	20	53
	64	91	37	25	21	22	16	10	11	52
	60	64	43	27	20	19	10	2	5	42

For the convenience of teachers an order of emphasis in the teaching of the Addition Facts is suggested in Table IV.

TABLE IV

Showing the

ADDITION FACTS

including zeros, in sum form in order of increasing difficulty, based on the average result of the three tests *

0 +0	2 +2	1 +4	1 +5	1 +3	1 +6	1 +8	1 +1	3 +1	3 +3
2 +1	4 +4	1 +2	8 +1	5 +5	1 +7	3 +0	4 +1	8 +0	2 +3
4 +0	4 +5	7 +0	2 +0	5 +0	7 +1	2 +5	3 +2	2 +4	5 +2
2 +6	3 +4	5 +1	6 +0	6 +1	7 +7	9 +1	1 +9	2 +7	2 +8
9 +0	2 +9	3 +5	4 +2	6 +6	7 +2	4 +3	5 +4	8 +2	3 +7
8 +8	5 +3	1 +0	6 +2	3 +6	3 +8	6 +3	9 +2	9 +9	4 +7
7 +3	4 +6	5 +6	6 +5	4 +8	6 +4	3 +9	8 +3	7 +4	8 +4
5 +7	6 +7	5 +9	9 +3	6 +9	8 +5	4 +9	7 +5	7 +6	8 +6
9 +4	9 +5	0 +5	0 +6	5 +8	6 +8	8 +9	7 +8	0 +4	0 +9
9 +6	8 +7	0 +2	7 +9	0 +7	9 +8	0 +8	0 +3	9 +7	0 +1

* Read in rows from left to right, not in column form.

SUMMARY OF FINDINGS

(1) The results are not random results.

(2) While there is improvement at each successive testing, the relative difficulty of the facts remains constant.

(3) The Addition Facts containing zero show less consistency than the others. The $\begin{array}{r} 0 \\ \underline{+x} \end{array}$ facts are definitely more difficult than the $\begin{array}{r} x \\ \underline{+0} \end{array}$ facts. The learning of the multiplication facts seems to be responsible for the deterioration in performance in these facts at the later tests.

APPENDIX I

9 +3	1 +3	9 +5	9 +2	4 +4	6 +4	5 +7	4 +7	5 +9	2 +1
3 +1	5 +3	7 +7	2 +7	3 +4	0 +7	6 +7	3 +5	1 +6	4 +2
6 +0	0 +2	8 +3	7 +4	4 +1	1 +1	0 +0	0 +9	9 +7	0 +6
3 +2	5 +8	1 +7	9 +1	9 +9	6 +3	2 +4	6 +5	6 +9	9 +4
7 +5	6 +1	8 +7	9 +0	4 +8	2 +8	0 +5	3 +8	0 +4	8 +8
3 +3	5 +5	8 +4	6 +6	8 +6	4 +6	7 +9	5 +1	2 +2	4 +5
7 +8	8 +5	1 +8	7 +1	2 +9	7 +2	7 +3	2 +6	2 +3	5 +0
3 +7	1 +5	8 +9	1 +4	2 +5	4 +0	6 +2	7 +0	9 +8	0 +3
9 +6	2 +0	8 +0	5 +4	4 +3	5 +6	0 +1	1 +0	3 +9	0 +8
1 +2	6 +8	7 +6	3 +0	1 +9	3 +6	4 +9	8 +1	5 +2	8 +2

SUBTRACTION FACTS

The results of the Subtraction Facts are presented in percentage form in Table V.

The results are not random results.

There is an improvement from test to test. With the exception of the facts containing zero, the orders of the facts are relatively constant from test to test.

As with the Addition Facts it will be observed that the facts containing zero are on the whole found to be more difficult at the later tests. This again may be due to the pupils having commenced the multiplication facts before the December 1945 test.

On the whole, as the size of the digits increases, the facts become more difficult. At the first test the pupils may have had much more practice with the smaller digits, but even at the third test there is no evidence that practice has affected the order. It seems clear that one element in the inherent difficulty of the fact is the size of the digits involved. It would be interesting to find out whether this persists right up to the adult stage.

The facts of the type $x-x$ are more difficult than might be expected.

Further evidence that each fact has a distinct degree of difficulty is given by the rank correlation between the results of the three tests. These are:

Between June 1945 and December 1945 . $r=0{\cdot}9115$
Between December 1945 and June 1946 . $r=0{\cdot}9664$
Between June 1945 and June 1946 . . $r=0{\cdot}8640$

It would therefore appear that not only has each fact its own inherent degree of difficulty, but that for each fact the degree of difficulty is becoming more definitive at the later tests.

RANKING ORDER

From the results in Table V the order of decreasing difficulty for the Subtraction Facts at each of the three tests was determined. These orders are shown in Table VI, together with the order as given by the average results of the three tests.

The fourth order, *i.e.* the one determined by the average results over the three tests, has been included to facilitate comparison. Since when one fact is down-graded another fact must be up-graded, it is found that in the case of almost all the facts not containing zero there is a very gradual up-grading, *e.g.*

	$6 - 5$
June 1945	63
December 1945 . . .	74
June 1946	79

TABLE V

Showing the percentage number who were successful in each of the

SUBTRACTION FACTS

June 1945 (*a*)

	−0	−1	−2	−3	−4	−5	−6	−7	−8	−9
0	**99·0**									
1	95·3	**92·5**								
2	97·8	92·7	**92·2**							
3	97·2	94·1	93·7	**92·4**						
4	97·3	95·0	94·6	93·9	**92·9**					
5	97·0	94·6	94·8	93·8	94·0	**93·0**				
6	96·9	94·1	93·0	93·7	92·2	92·5	**92·8**			
7	97·5	94·3	91·7	92·5	91·7	92·3	92·7	**92·8**		
8	97·8	95·1	92·7	92·0	93·1	91·3	89·2	91·7	**92·7**	
9	96·5	94·0	90·3	91·7	91·3	90·6	88·4	85·8	89·4	**92·2**
10		92·2	89·5	90·0	89·2	92·4	88·6	86·4	87·7	89·5
11			85·7	82·4	81·6	84·8	84·4	82·4	82·9	83·4
12				82·8	82·6	83·2	86·1	81·7	83·7	82·7
13					80·4	81·2	79·1	81·9	81·1	80·4
14						79·7	79·3	80·2	80·9	80·7
15							76·4	78·3	78·5	76·8
16								76·4	76·9	73·7
17									76·5	77·4
18										78·7

Mean of all scores =88·47
Standard Deviation of scores = 16·69

December 1945 (*b*)

	−0	−1	−2	−3	−4	−5	−6	−7	−8	−9
0	**99·2**									
1	93·1	**93·3**								
2	94·8	95·2	**93·4**							
3	93·9	97·2	96·8	**92·9**						
4	95·1	97·2	96·9	95·7	**93·2**					
5	94·5	97·1	96·4	95·7	96·6	**93·2**				
6	94·3	96·4	96·0	95·9	94·8	94·9	**93·6**			
7	95·4	96·4	95·3	94·9	93·9	94·5	95·4	**93·2**		
8	95·3	97·1	95·8	94·1	95·5	94·0	93·0	94·7	**94·2**	
9	95·0	97·7	94·2	93·5	94·0	93·3	91·4	91·1	94·6	**92·7**
10		93·4	89·3	90·0	89·3	93·8	90·7	89·3	90·6	91·9
11			87·2	86·0	85·3	87·3	88·7	84·6	86·1	86·0
12				85·8	86·8	86·1	89·0	85·1	85·5	85·5
13					81·7	84·2	82·6	85·0	82·3	82·9
14						82·8	83·2	86·6	84·0	82·7
15							80·0	79·6	81·6	80·2
16								79·5	84·9	79·0
17									80·5	80·2
18										84·6

Mean of all scores =90·66
Standard Deviation of scores=15·19

June 1946

(*c*)

	−0	−1	−2	−3	−4	−5	−6	−7	−8	−9
0	**99·4**									
1	95·0	**94·4**								
2	95·7	95·5	**93·9**							
3	95·5	97·5	96·8	**94·1**						
4	95·4	97·5	98·2	97·1	**95·1**					
5	94·7	97·9	96·9	96·9	96·7	**95·2**				
6	96·5	97·2	96·8	97·4	96·1	96·5	**94·3**			
7	95·2	97·0	96·4	95·0	96·1	95·3	96·7	**95·2**		
8	95·8	96·6	97·0	95·4	96·5	95·7	95·1	96·7	**94·7**	
9	95·3	97·7	95·0	95·8	95·4	95·8	94·2	94·7	95·8	**94·1**
10		95·3	93·7	92·6	93·4	93·3	94·4	93·5	94·3	94·6
11			91·8	89·0	88·7	90·5	92·1	89·3	90·1	90·8
12				90·9	90·8	89·2	93·5	90·1	91·7	90·2
13					88·1	87·4	88·2	90·4	88·6	87·7
14						87·6	89·3	91·7	88·6	87·0
15							86·5	88·8	87·2	84·8
16								86·9	90·2	85·2
17									85·6	86·9
18										88·9

Mean of all scores = 93·36
Standard Deviation of scores = 12·56

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TABLE VI

Showing the ranking order in decreasing difficulty of the various

SUBTRACTION FACTS

according to June 1945, December 1945, June 1946, and average of three tests

	−0	−1	−2	−3	−4	−5	−6	−7	−8	−9
0	100 100 100 100									
1	91 49 53 70	63 53 47 53								
2	98 72 69 93	67 78 67 70	63 56 41 49							
3	95 60 67 80	83 97 95 94	77 93 86 85	61 47 42 48						
4	96 77 64 87	90 97 95 99	86 94 99 96	80 84 92 82	56 50 56 57					
5	94 68 50 79	87 95 98 96	88 89 88 91	79 84 88 80	81 92 82 85	74 50 58 61				
6	93 67 79 87	83 89 93 87	74 88 86 78	77 87 94 84	57 72 75 68	63 74 79 73	72 58 45 54			

Correlations:

June 1945 and December 1945 =·985
December 1945 and June 1946 =·964

TABLE VI—*continued*

	−0	−1	−2	−3	−4	−5	−6	−7	−8	−9
7	97 81 58 91	85 89 90 87	51 79 78 70	63 74 53 66	51 60 75 63	60 68 61 65	67 81 82 75	72 50 58 57		
8	98 79 71 94	67 81 82 75	67 86 90 74	55 64 64 61	75 84 79 77	49 62 69 57	41 48 56 46	51 71 82 68	67 65 50 63	
9	92 76 61 82	81 99 97 96	47 65 53 49	51 56 71 57	49 62 64 54	48 53 71 49	39 44 44 44	35 43 50 38	43 70 71 52	57 46 42 47
10		57 55 61 54	44 37 40 40	46 40 36 41	41 37 37 39	61 59 77 67	40 42 47 43	37 37 38 37	38 41 45 41	45 45 49 45
11			34 33 34 34	24 27 19 23	21 23 16 20	34 34 28 33	33 35 35 35	25 18 21 21	29 29 23 28	31 27 29 30
12				28 26 31 29	26 32 29 30	30 29 20 26	36 36 38 36	22 22 23 22	32 24 32 32	27 24 25 25
13					15 9 12 9	20 17 9 18	10 11 13 13	23 21 27 23	19 10 14 15	15 14 11 14
14						13 13 10 9	11 15 21 12	14 31 32 26	18 16 14 19	17 12 7 11

TABLE VI—*continued*

	−0	−1	−2	−3	−4	−5	−6	−7	−8	−9
15							2 4 4 5	7 3 17 7	8 8 8 8	5 5 1 2
16								2 2 5 3	12 20 25 15	1 1 2 1
17									4 7 3 3	6 5 5 6
18										9 18 18 17

TABLE VII

Showing the

SUBTRACTION FACTS

in sum form in order of increasing difficulty, based on the average result of the three tests *

0 -0	4 -1	4 -2	5 -1	9 -1	8 -0	3 -1	8 -1	2 -0	5 -2
7 -0	4 -0	6 -0	6 -1	7 -1	3 -2	5 -4	6 -3	9 -0	3 -0
4 -3	5 -0	5 -3	6 -2	8 -2	8 -4	7 -6	6 -5	1 -0	2 -1
7 -2	6 -4	8 -7	7 -3	7 -5	10 - 5	7 -4	5 -5	8 -3	4 -4
7 -7	8 -5	9 -3	10 - 1	6 -6	8 -8	9 -4	1 -1	9 -5	9 -8
2 -2	3 -3	9 -2	9 -9	8 -6	10 - 9	9 -6	10 - 6	10 - 8	10 - 2
10 - 3	10 - 4	9 - 7	10 - 7	12 - 6	11 - 6	11 - 2	11 - 5	12 - 8	11 - 9
12 - 4	12 - 3	11 - 8	12 - 5	12 - 9	14 - 7	11 - 3	13 - 7	12 - 7	11 - 7
11 - 4	14 - 8	13 - 5	13 - 8	16 - 8	18 - 9	14 - 6	13 - 9	13 - 4	14 - 9
13 - 6	15 - 8	15 - 7	17 - 9	15 - 6	16 - 7	17 - 8	15 - 9	16 - 9	14 - 5

---

* Read in rows from left to right, not in column.

SUMMARY OF FINDINGS

(1) The results are not random results, *i.e.* the failures are not due to chance causes.

(2) Each Subtraction Fact has its own degree of difficulty at a particular stage but the difficulty is less at later stages.

(3) As the size of the digits increases, the facts become more difficult.

(4) The facts containing a zero, *i.e.* of the type $x - 0$, are more difficult than those of the type $x - 1$, $x - 2$, etc. at the stage of the third test. The learning of the multiplication and division facts seems to be responsible for this deterioration in performance.

(5) Facts of the type $x - x$ are more difficult than might have been expected.

APPENDIX II

10 - 6	12 - 3	4 -3	14 - 5	11 - 2	8 -4	10 - 4	12 - 7	11 - 7	14 - 9
3 -1	4 -1	8 -3	14 - 7	9 -7	7 -4	7 -7	13 - 7	8 -5	7 -6
6 -2	6 -0	2 -2	11 - 3	11 - 4	5 -1	2 -1	0 -0	9 -9	16 - 7
6 -6	5 -2	13 - 8	8 -7	10 - 1	18 - 9	9 -3	6 -4	11 - 5	15 - 9
13 - 4	12 - 5	7 -1	15 - 7	9 -0	12 - 8	10 - 8	5 -5	11 - 8	4 -4
16 - 8	6 -3	10 - 5	12 - 4	12 - 6	14 - 6	16 - 9	6 -1	4 -2	9 -5
15 - 8	13 - 5	9 -8	8 -1	11 - 9	9 -2	10 - 3	8 -6	5 -3	5 -0
10 - 7	6 -5	17 - 9	5 -4	7 -5	4 -0	8 -2	7 -0	17 - 8	3 -3
10 - 9	15 - 6	2 -0	8 -0	9 -4	7 -3	11 - 6	1 -1	1 -0	12 - 9
8 -8	3 -2	14 - 8	13 - 6	3 -0	9 -1	13 - 9	9 -6	7 -2	10 - 2

ADDITION AND SUBTRACTION PROCESSES

Construction of Tests

To enable teachers to construct analogous sets of tests to those employed in this investigation, it has been deemed advisable to indicate the plan followed in their preparation.

Addition Processes

In the Addition Process zeros may be inserted in any of the rows, and in any of the columns except the first. One, two or more zeros may also be incorporated in any column according to the number of rows, and the difficulty of the zeros may be increased if two zeros in a column succeed each other. Broken lines may be introduced into any row of an addition sum of more than one column. With three columns the broken line may consist of the unit figure only or of both the unit and tens digits. Carrying may or may not occur in any addition sum of more than one column and of a reasonable number of rows. Bridging the tens and/or the twenties, thirties, etc. may be required in any column of which the sum is more than 10 or 20, etc. These variations may be combined in several ways, giving rise to a series of examples only limited by the number of rows or columns agreed upon.

The Addition Facts are examples of the Addition Process with two rows and one column. A zero may occur in either the top or bottom row, thus giving rise to 2 possibilities. A broken line cannot occur with an example of one column only—0 possibilities. Carrying cannot occur with a one-column two-row addition—0 possibilities. Bridging the ten may or may not occur, but bridging twenty is impossible—2 possibilities.

The possibilities with three and four rows can in similar fashion be easily enumerated.

With two-column examples zeros can only occur in the units column, but rows of varying lengths can occur in the second column. Carrying may or may not take place. Bridging ten may or may not occur in either or both columns.

This procedure may be followed for all the tests up to three columns and four rows—the limits set in this investigation.

Without taking into account the possible combinations of zeros with broken lines and bridging, some hundreds of examples can be prepared.

SUBTRACTION PROCESSES

Only two rows are possible in subtraction, but any number of columns—tens, hundreds, thousands, etc. up to the limit imposed by the investigation, which in this case was hundreds.

In the Subtraction Facts the top line may consist of units or tens (up to 18) but the bottom line only up to 9. The units digit of the top line may be a zero but the tens figure cannot be. In the bottom line a zero may occur. A broken line is inevitable when the top line exceeds 9. There is no borrowing, but dealing with tens may be necessary. There are at least 5 possibilities in the facts.

With tens in the top line the units figure may be a zero, as also in the bottom line. A broken line may occur; borrowing may or may not be necessary. Dealing with tens may be involved.

With hundreds in the top line and in the bottom line, zeros may occur apart from broken rows in twelve different positions. The broken line may take the form of tens or units in the bottom line. Borrowing may not occur or may occur once or twice. Handling tens may also be required twice. The alternatives would be too numerous for any single test.

SUBJECTS

The same group of pupils was examined at each of the three tests. The number in the group was less at the later tests on account of removals, etc. At each date all the pupils in the group were examined in the Facts and in the Processes.

In reducing the data from the tests of Addition and Subtraction Processes no account was taken of age, sex or time taken.

Addition Processes

The results of the tests in the Addition Facts were presented in Table I, and for each combination the percentage number correct has been determined. These percentages may be referred to as probability-values or *p*-values. Thus, in Table I (*a*) it is found that

Percentage correct in the fact 4+5, *i.e.* $\begin{array}{r}4\\ +5\\ \hline\end{array} = 96{\cdot}7$

Hence

Probability of success in the fact $4+5 \quad = \quad {\cdot}967$

or

$$p_{4+5} = {\cdot}967$$

This gives a more precise meaning to the result, and is essential in the analysis of the Addition and Subtraction Processes into their component facts.

As the *p*-values are not random values, as has already been shown, there must be inherent in the mental operation involved in each fact a property which determines the degree of difficulty of the fact, and this in turn will be the main factor in fixing the *p*-value. If this is the case, are the *p*-values constant? For example, does a group of pupils find the fact 4+5 equally difficult in all situations?

$$\begin{array}{cccc} & & & 1\\ 4 & 4 & 4 & 4\\ 0 & 2 & 4 & 3\\ +5 & +3 & +5 & +2\\ \hline \end{array}$$

The fact 4+5, *i.e.* $\begin{array}{r}4\\ +5\\ \hline\end{array}$ occurs in all these processes. Will pupils at a given stage find it equally difficult in the varying situations in which it occurs in these processes?

If each fact has its own distinct *p*-value, does this *p*-value alter with the mental growth of the group? If so, has each fact a maximum p-value, and at what stage in the school course is this reached?

Each Addition Process can be analysed into two or more Addition Facts, and may or may not include 'carrying' operations. Now the probability of the Addition Facts composed of digits up to 9 is known when these facts are presented as unit operations. Will the same probabilities hold when several of these Addition Facts are combined in an Addition Process? Consider the process

$$\begin{array}{r} 9 \\ 0 \\ +6 \\ \hline \end{array}$$

This involves two Addition Facts, namely

$$\left.\begin{array}{l} (1)\ 0+6 \text{ with a } p\text{-value} = \cdot 941 \\ (2)\ 9+6 \text{ with a } p\text{-value} = \cdot 900 \end{array}\right\} \text{June 1945 values}$$

If these probabilities are inherent in the Addition Facts, no matter in what situation they occur, then the probability of success in the Addition Process will be the product of these probabilities.

Hence probability of success in process $= \cdot 941 \times \cdot 900$
$= \cdot 846$

Now the p-value for the process in June 1945 is ·901.

$$\text{Hence}\quad \frac{\text{actual } p\text{-value}}{\text{theoretical } p\text{-value}} = \frac{\cdot 901}{\cdot 846} = 1\cdot 064$$

Thus, on the average, pupils would appear to be more successful in the Addition Facts when these are combined in the Addition Process.

The above process does not involve a carrying figure, but here is one that does:

$$\begin{array}{r} 74 \\ 6 \\ +85 \\ \hline \end{array}$$

This process includes the following facts:

(1) $6+\ 5$ with a p-value $= \cdot 943$
(2) $4+11$ with a p-value (say) $= \cdot 900$

This last value is obtained by extrapolation from Table I. Next, there is a 'carrying' operation the probability of success in which is not known:

(3) $8+1$ with a p-value $= \cdot 963$
(4) $7+9$ with a p-value $= \cdot 900$

The probability of success in all four facts will be

$$\cdot 943 \times \cdot 900 \times \cdot 963 \times \cdot 900 = \cdot 735$$

The p-value obtained for this process in the June test is ·618. The difference in these two values is due to the 'carrying' operation:

$$p\text{-value for the carrying operation} = \frac{\cdot 618}{\cdot 735} = \cdot 840$$

Has the carrying operation the same chance of success in all situations? This can be tested by analysing other processes.

Unfortunately, the number of Addition Processes in the test which, at present, can be analysed is limited, as the p-values for the Addition Facts which include numbers beyond 9 are not known. Facts of the type $+10$ and $+11$ are near enough the range of the known p-values to enable one to estimate their p-values by extrapolation.

Here is the analysis of a process involving a carrying operation:

$$\begin{array}{r} 346 \\ 437 \\ \underline{+214} \end{array}$$

(1) $7+4$ with a p-value $= \cdot 936$
(2) $6+11$ with a p-value $= \cdot 900$.

Then the operation of carrying 1:

(3) $1+1$ with a p-value $= \cdot 973$
(4) $3+2$ „ „ $= \cdot 960$
(5) $4+5$ „ „ $= \cdot 967$
(6) $4+2$ „ „ $= \cdot 953$
(7) $3+6$ „ „ $= \cdot 941$

The probability of success in all these seven Addition Facts $= \cdot 682$.

The actual probability of success obtained from the results of the June 1945 Addition Process test $= \cdot 581$. Hence the probability of success in the carrying operation

$$= \frac{\cdot 581}{\cdot 682} = \cdot 851$$

It is very remarkable that, with all the unit complexities involved, the p-value for the carrying operation should be so nearly the same in the above two processes. In these two processes the carrying figure is 1, and it is carried from the first to the second column. It would appear that at the particular stage of development represented by the group of pupils at the June 1945 test this residual p-value is about ·84.

The p-values used in the above analyses are for June 1945 tests only. In the analysis of the processes for the other tests the p-values for the Addition Facts given by the December 1945 and June 1946 tests are used in turn. The results of the analyses of nine processes are shown in Table IX. In each case the residual p-value is the ratio of the actual p-value to the calculated p-value.

Examination of Table IX reveals the following points:

(1) *Processes with no carrying operations*

(*a*) For these six processes the residual p-values at each of the tests are, with one exception, greater than unity.

(*b*) For these processes there is a definite increase in the residual p-value from test to test.

This brings to light the very important fact that pupils find less difficulty with the Addition Facts when these are presented in combination in the Processes than when they are presented singly. The explanation may be that at the school stages represented by the group at the time of the later tests the pupils are in process of discarding the early visual and/or auditory aids to addition and are proceeding to add in a purely mechanical way. In column addition they are now tending to do the unit operations mechanically, but in the simple addition of two numbers, as in an addition fact, force of habit makes them revert to the earlier visual and/or auditory methods.

(2) *Processes with one carrying operation*

The three processes which have been analysed give very definite residual *p*-values for each of the three tests, and these values increase steadily from test to test. These values show clearly that the incidence of a carrying operation in an addition process increases the difficulty of the process.

These residual *p*-values do not by themselves indicate the effect of the carrying operation. They must be examined along with the residual *p*-values for the processes containing no carrying operation; thus, to obtain an absolute measure of the effect of the carrying operation, the residual *p*-values for the last three processes must be reduced in proportion as the effect of transition to mechanical addition increases. The fact that the residual *p*-values for the June 1946 test are unity cannot, therefore, be interpreted as meaning that pupils at the time of this test were 100 per cent. successful with the carrying operation.

Note.—The number of processes which can be analysed is too small to justify any further analysis or the assignment of exact numerical values.

The examples set, together with the percentages of pupils correct in the June 1945, December 1945 and June 1946 tests in the Addition Processes are presented in Table VIII.

TABLE VIII

Showing percentage of children correct in, wrong in, and not attempting various

ADDITION PROCESSES

at end of Infant Division in June 1945, in December 1945 and in June 1946

(No. = 1,300 in June 1945, 1,114 in December 1945
and 1,045 in June 1946)

Process	Percentage correct			Percentage wrong			Percentage not attempted		
	June 1945	Dec. 1945	June 1946	June 1945	Dec. 1945	June 1946	June 1945	Dec. 1945	June 1946
4 + 1 + 3	96·5	97·4	98·7	3·4	2·6	1·3	..	..	..
5 + 0 + 4	95·4	96·9	97·8	4·5	3·1	2·2	..	..	..
5 + 4 + 7	89·0	91·3	94·5	11·0	8·7	5·3	..	..	·2
9 + 0 + 6	90·1	93·0	96·7	9·8	7·0	3·1	..	..	·2
6 + 8 + 9	81·3	84·7	91·1	18·6	15·0	8·7	..	·3	·2
2 + 0 + 6 + 1	92·8	96·8	98·0	7·1	3·2	2·0	..	..	..
4 + 2 + 7 + 3	88·3	91·1	94·9	11·6	8·8	5·1	..	·1	..
6 + 8 + 4 + 0	85·9	91·7	94·2	14·0	8·2	5·8	..	·1	..
5 + 7 + 9 + 8	72·1	77·8	85·2	27·6	21·6	14·7	·1	·6	·1
9 + 7 + 0 + 8	75·4	79·3	88·3	24·3	20·1	11·7	·1	·6	..
19 + 55 + 17	49·0	62·7	75·7	46·7	37·0	24·2	4·2	·3	·1
74 + 6 + 85	61·8	76·6	82·9	33·7	23·1	17·0	4·3	·3	·1
42 + 26 + 20 + 31	77·3	86·4	89·8	19·9	13·2	10·0	2·6	·4	·2
28 + 59 + 65 + 37	43·6	60·7	71·1	51·3	38·3	28·6	5·0	1·0	·3
346 + 437 + 214	58·1	71·9	79·2	36·5	26·6	20·5	5·3	1·5	·3
516 + 239 + 214	58·6	72·4	80·5	35·6	26·7	19·1	5·6	·9	·4
593 + 90 + 125	49·4	69·6	78·5	44·6	29·3	21·2	5·9	1·1	·3
816 + 372 + 611	70·6	82·5	87·9	25·2	16·1	11·7	4·0	1·4	·4
3 + 71 + 994	53·2	73·6	84·3	40·8	24·9	15·4	5·9	1·5	·3
143 + 290 + 487	50·4	68·1	79·1	43·2	30·2	20·3	6·3	1·7	·6

Note.—As the pupils were expected to add upwards, the processes in this Table should be read thus 4 + 1 + 3 =

$$\begin{array}{r} 4 \\ 1 \\ +3 \\ \hline \end{array}$$

TABLE IX

Showing the calculated probability of success in certain of the Processes, together with their Residual Probabilities

$$\text{Residual probability} = \frac{\text{Actual probability}}{\text{Calculated probability}}$$

The actual probabilities can be calculated from Table VIII

Process	No. of Facts	Carrying Ops.	June 1945		December 1945		June 1946	
			Calculated p	Residual p	Calculated p	Residual p	Calculated p	Residual p
4 1 +3	2	0	·948	1·018	·964	1·010	·964	1·023
5 0 +4	2	0	·892	1·069	·882	1·099	·891	1·097
9 0 +6	2	0	·847	1·064	·843	1·103	·871	1·110
2 0 6 +1	3	0	·857	1·082	·858	1·127	·862	1·136
42 26 20 +31	6	0	·739	1·046	·768	1·126	·715	1·257
816 372 +611	6	0	·727	·972	·804	1·025	·850	1·034
74 6 +85	4	1	·735	·840	·801	·955	·831	·998
516 239 +214	7	1	·700	·837	·749	·968	·792	1·016
346 437 +214	7	1	·682	·852	·747	·962	·790	1·003

Summary of Findings

(1) At the December 1945 and June 1946 tests there is evidence of a transition stage in addition from being a mental process involving visual and/or auditory aids to being a purely mechanical operation. This transition to mechanical addition lessens the difficulty.

(2) The carrying operation increases the difficulty of a process, but the degree of difficulty is less at later stages.

SUBTRACTION PROCESSES

Table V presented the results of the Subtraction Facts for the three tests. These are given as percentages, but as with the Addition Processes these will be referred to as probability-values or p-values. Thus from Table V:

Percentage number correct in the fact $8 - 3 = 92{\cdot}0$

Hence

Probability of success in the fact $8 - 3 = {\cdot}920$

This means that if 1,000 pupils at the given stage try this fact, then we can expect 920 of them to be successful.

This gives a more precise meaning to the result and, as will be seen later, is essential in the analysis of the processes.

Each Subtraction Process can be analysed into:

(*a*) Two or more Subtraction Facts. The probability of success in each fact at each of the three tests is known.

(*b*) One or more borrowing operations, occurring in the first, the second, or in both first and second columns.

b_1 = borrowing in the first column.
b_2 = borrowing in the second column.

In addition, the following conditions may affect the chances of success:

(*c*) A zero in the top line Z
(*d*) A zero in the bottom line z
(*e*) A missing digit, *i.e.* where the number of digits in the bottom line is less than the number in the top line M

The symbols shown above are used to describe a process, thus:

$$\begin{array}{r} 603 \\ -206 \\ \hline \end{array} = b_1b_2Zz \qquad \begin{array}{r} 43 \\ -\ 6 \\ \hline \end{array} = b_1M$$

Now for each process the probability of success in all the Subtraction Facts together can be calculated. This combined probability will not, however, agree with the actual probability of success in the process as given by actual results. The difference in calculated and observed probabilities will be due to the influence of the other factors mentioned above. The actual probability of success is equal to the product of the probabilities of the facts and factors involved; hence

$$\frac{\text{Actual probability of success}}{\text{Probability of success in facts only}} = \begin{array}{c}\text{Probability of success} \\ \text{in factors only}\end{array}$$

As in the Addition Processes this ratio is called the *Residual Probability*.

Consider the process $\begin{array}{r} 90 \\ -\ 8 \\ \hline \end{array} = b_1ZM$

This involves the facts:

10 – 8 with a probability of ·877
9 – 1 with a probability of ·940

at June 1945.

Probability of success in both = ·877 × ·940
= ·824

∴ Actual probability of success in the process as given by the results of the test = ·358

Hence, residual probability = .358/·825
= ·434

This means that the probability of success with a borrowing operation in the first column, a zero in the top line, and a missing digit is ·434, quite apart from the probability of being successful in the facts.

·434 is, in fact, the product of the probabilities for b_1, Z and M.

A similar analysis was made of each of the processes. The results are summarised in Table XI.

An examination of the results in Table XI gives some indication as to how the chances of success in the processes were affected by the secondary factors of

Borrowing in the first column	b_1
Borrowing in the second column	b_2
Zero digit in the top line	Z
Zero digit in the bottom line	z
Missing digit	M

In Table XII the results given in Table XI are further summarised, *e.g.* there are three processes of the type b_1b_2Z, and in Table XII the *average* residual probability of these three processes is given.

Finally, in Table XIII is shown the probability of success associated with each separate factor.

Table XIII, it is suggested, contains some important information which has not hitherto been ascertained. When the simple arithmetical facts of addition and subtraction are combined into a process, then certain factors are introduced which may or may not have an effect on success in the process. The actual effects which such factors have on success at the three tests are given.

COMMENTS ON TABLE XIII

(*a*) For all factors the values of the residual probabilities are unsteady at the June 1945 test, but appear to have reached a definite steady value at the subsequent tests.

(*b*) The values for M and *z* are high enough to justify the conclusion that a missing digit and a zero in the bottom line do not add to the difficulty of the processes.

(*c*) A zero in the top line reduces the chance of success by 20%, *i.e.* with a group of 1,000 pupils who can do the facts in a process perfectly, if there is a zero in the top line only 800 will be successful.

(*d*) A borrowing operation in the first column reduces the chance of success by 20%, and a borrowing operation in the second column also reduces the chance of success by 20%. It is interesting to note that borrowing operations in both the first and second columns have an exactly similar effect. This would seem to imply that if a pupil is successful with a borrowing operation in the first column, he will also be successful with a borrowing operation in the second column of the same process.

(*e*) Now that the probabilities of all the facts and factors which are involved in any subtraction process are known, it is possible to predict, with a fair degree of accuracy, the percentage number who will be successful in any given subtraction process when it is set to a group similar to the group used for the experiment.

The examples set, together with the percentages of pupils correct in the June 1945, December 1945 and June 1946 tests in the Subtraction Processes, are presented in Table X.

TABLE X

Showing percentage of children correct in, wrong in, and not attempting various

SUBTRACTION PROCESSES

at end of Infant Division in June 1945, in December 1945 and in June 1946

(No. = 1,306 in June 1945, 1,108 in December 1945 and 1,044 in June 1946)

Process	Percentage correct			Percentage wrong			Percentage not attempted		
	June 1945	Dec. 1945	June 1946	June 1945	Dec. 1945	June 1946	June 1945	Dec. 1945	June 1946
98 − 43	86·1	90·1	93·7	13·6	9·7	6·2	·4	·09	..
47 − 20	89·6	94·0	93·7	9·9	5·9	6·2	·5	..	..
68 − 4	78·4	82·1	87·5	21·3	17·7	12·4	·3	·18	·09
64 − 37	33·8	68·4	70·6	39·5	31·5	25·3	26·6	·09	·09
50 − 27	31·5	55·9	66·2	41·0	44·1	33·7	27·5	..	·09
43 − 6	33·6	59·5	70·4	38·1	40·2	29·5	28·4	·18	·09
90 − 8	35·8	55·5	67·5	37·4	44·4	32·1	26·7	·09	·29
528 − 415	81·0	89·1	91·8	16·2	10·6	8·1	2·9	·18	·09
954 − 130	84·3	90·7	93·1	12·9	9·0	6·7	2·8	·27	·09
927 − 304	80·9	87·9	91·6	16·0	11·8	8·2	3·1	·18	·09
563 − 53	70·0	77·8	84·8	24·7	21·8	15·0	5·2	·36	·09
576 − 158	29·4	63·4	68·8	39·3	35·8	30·7	31·3	·72	·19
648 − 155	25·7	56·9	65·6	44·2	42·4	34·1	30·1	·63	·29
340 − 134	25·7	51·2	61·6	44·0	48·0	38·2	30·3	·72	·19
708 − 152	25·6	49·8	59·9	44·2	49·5	39·8	30·2	·63	·29
393 − 58	24·6	52·9	63·1	43·6	46·1	36·5	31·9	·90	·38
625 − 85	28·0	55·9	66·3	40·8	43·1	33·3	31·2	·99	·29
230 − 7	25·1	48·8	61·0	43·6	50·1	38·6	31·3	1·10	·38
900 − 70	27·9	52·2	63·8	40·8	46·6	35·9	31·3	1·26	·29
934 − 465	21·1	50·2	60·2	46·4	48·6	39·4	32·5	1·17	·38
920 − 146	19·8	45·0	55·5	47·6	53·8	44·1	32·5	1·17	·48
902 − 786	11·8	49·1	59·1	44·3	49·2	40·3	33·9	1·71	·57
716 − 79	22·3	47·6	59·9	43·4	50·8	39·6	34·3	1·53	·38
603 − 206	17·6	41·3	56·2	48·6	57·1	43·4	33·8	1·53	·38
920 − 68	21·6	45·0	57·8	44·4	53·2	41·8	34·0	1·71	·48
701 − 299	21·1	49·2	60·1	44·3	49·1	39·3	34·5	1·71	·57
600 − 53	21·7	45·6	59·4	44·1	52·5	39·8	34·2	1·89	·66

TABLE XI

Showing the Residual Probabilities for each of the
SUBTRACTION PROCESSES
at each of the tests

Process	Type	Facts	Residual Probabilities		
			June 1945	Dec. 1945	June 1946
98 −43	..	8−3 9−4	1·025	1·019	1·030
47 −20	z	7−0 4−2	·973	1·017	1·006
528 −415	..	8−5 2−1 5−4	1·018	1·031	1·038
954 −304	z	4−0 5−3 9−1	·983	1·020	1·032
64 −37	b_1	14−7 1+3 6−4	·467	·851	·860
50 −27	b_1Z	10−7 1+2 5−3	·400	·668	·745
576 −158	b_1	16−8 1+5 7−6 5−1	·448	·817	·818
340 −134	b_1Z	10−4 1+3 4−4 3−1	·337	·651	·722
708 −152	b_2Z	8−2 10−5 1+1 7−2	·334	·590	·678

TABLE XI—*continued*

Process	Type	Facts	Residual Probabilities		
			June 1945	Dec. 1945	June 1946
648 −155	b_2	8 − 5 14 − 5 1 + 1 6 − 2	·390	·773	·824
934 −465	b_1b_2	14 − 5 1 + 6 13 − 7 1 + 4 9 − 5	·375	·790	·821
920 −146	b_1b_2Z	10 − 6 1 + 4 12 − 5 1 + 1 9 − 2	·312	·631	·719
902 −786	b_1b_2Z	12 − 6 1 + 8 10 − 9 1 + 7 9 − 8	·181	·655	·726
603 −206	b_1b_2Zz	13 − 6 1 + 0 10 − 1 1 + 2 6 − 3	·275	·603	·721
701 −299	b_1b_2Z	11 − 9 1 + 9 10 − 10 1 + 2 7 − 3	·321	·691	·772
68 − 4	M	8 − 4 6 − 0	·869	·912	·940

TABLE XI—*continued*

Process	Type	Facts	Residual Probabilities		
			June 1945	Dec. 1945	June 1946
43 − 6	b_1M	13 − 6 4 − 1	·447	·741	·819
90 − 8	b_1ZM	10 − 8 9 − 1	·434	·627	·733
563 − 53	M	3 − 3 6 − 5 5 − 0	·844	·934	·987
393 − 58	b_1M	13 − 8 1 + 5 9 − 6 3 − 0	·363	·759	·805
625 − 85	b_2M	5 − 5 12 − 8 1 + 0 6 − 1	·397	·769	·806
900 − 70	b_2ZZz	0 − 0 10 − 7 1 + 0 9 − 1	·360	·635	·724
716 − 79	b_1b_2M	16 − 9 1 + 7 11 − 8 1 + 0 7 − 1	·412	·781	·848
920 − 68	b_1b_2ZM	10 − 8 1 + 6 12 − 7 1 + 0 9 − 1	·341	·641	·730
600 − 53	b_1b_2ZZM	10 − 3 1 + 5 10 − 6 6 − 1	·308	·619	·731

TABLE XII

Showing average Residual Probabilities

Type	No. of Processes	Average Residual Probabilities		
		June 1945	Dec. 1945	June 1946
	2	1·022	1·025	1·034
z	3	·980	1·031	1·026
b_1	2	·457	·834	·839
b_1Z	2	·368	·659	·733
b_2	1	·390	·773	·824
b_2Z	1	·335	·590	·678
b_1b_2	1	·375	·790	·821
b_1b_2Z	3	·271	·659	·739
b_1b_2Zz	1	·275	·603	·721
M	2	·856	·923	·963
b_1M	2	·405	·750	·812
b_1ZM	1	·434	·627	·733
b_2M	1	·397	·769	·806
b_2ZZzM	1	·360	·635	·724
b_1b_2M	1	·412	·781	·848
b_1b_2ZM	1	·341	·641	·730
b_1b_2ZZM	1	·308	·619	·731

TABLE XIII

Showing the average Residual Probabilities for each of the factors in

SUBTRACTION PROCESSES

which affect success

Factor	How obtained	Residual Probabilities		
		June 1945	Dec. 1945	June 1946
Z	$\frac{b_1 Z}{b_1}$	·805	·791	·874
	$\frac{b_2 Z}{b_2}$	·859	·763	·823
	$\frac{b_1 b_2 Z}{b_1 b_2}$	·723	·834	·900
	$\frac{b_1 b_2 ZM}{b_1 b_2 M}$	·828	·821	·861
	Averages	$\boxed{\cdot 804}$	$\boxed{\cdot 802}$	$\boxed{\cdot 864}$

TABLE XIII—*continued*

Factor	How obtained	Residual Probabilities		
		June 1945	Dec. 1945	June 1946
M	$\frac{b_1M}{b_1}$	·886	·899	·968
	$\frac{b_1ZM}{b_1Z}$	1·180	·952	1·000
	$\frac{b_2M}{b_2}$	1·020	·995	·978
	$\frac{b_1b_2M}{b_1b_2}$	1·100	·987	1·033
	$\frac{b_1b_2ZM}{b_1b_2Z}$	1·260	·973	·988
	Averages	$\boxed{1\cdot 089}$	$\boxed{\cdot 961}$	$\boxed{\cdot 988}$
z	direct	·959	1·006	·993
	$\frac{b_1b_2z}{b_1b_2}$	1·015	·915	·976
	Averages	$\boxed{\cdot 987}$	$\boxed{\cdot 960}$	$\boxed{\cdot 984}$
b_1	direct	$\boxed{\cdot 457}$	$\boxed{\cdot 834}$	$\boxed{\cdot 839}$
b_2	direct	$\boxed{\cdot 390}$	$\boxed{\cdot 773}$	$\boxed{\cdot 824}$
b_1b_2	direct	$\boxed{\cdot 375}$	$\boxed{\cdot 790}$	$\boxed{\cdot 821}$

Summary of Findings

(1) The facts retain their inherent degrees of difficulty when combined in Subtraction Processes.

(2) The incidence of borrowing operations in either the first or the second column or in both columns reduces the chance of success in the process by 20%.

(3) A zero in the top line also reduces the chance of success by 20%.

(4) A zero in the bottom line does not affect the chance of success.

(5) A missing digit, *i.e.* when the number of digits in the bottom line is less than the number of digits in the top line, does not alter the chance of success.

(6) Any Subtraction Process can be analysed into its component facts and incidental factors which affect success. As the probabilities of success for these facts and factors are now known the probability of success in the process can be estimated. It is thus possible to make up Subtraction Processes with any given degree of difficulty, *e.g.* a process in which we can be fairly certain that 70 per cent. will pass. A complete test can, therefore, be constructed containing processes with prearranged degrees of difficulty and which will give any required distribution of marks. This will be invaluable in the construction of standardised attainment tests in arithmetic.

RECOMMENDATIONS

General

The Panel advises that the equation form of presenting the basic Arithmetic Facts and Processes should be abandoned in all teaching in, and textbooks for, the Primary school.

The signs of the Processes should be inserted in all examples.

Addition Facts

As the facts $\begin{array}{r}0\\+x\\\hline\end{array}$ prove more difficult than the $\begin{array}{r}x\\+0\\\hline\end{array}$ facts, the $\begin{array}{r}0\\+x\\\hline\end{array}$ facts should have special consideration.

To avoid confusion between the addition and the multiplication facts, special care should be taken with the zero addition combinations when multiplication is introduced.

Subtraction Facts

As facts of the $x - x$ have been found to be more difficult than might have been expected statistically, special attention should be given to them.

Addition and Subtraction Processes

In preparing tests teachers should introduce only one difficulty at a time, preferably in order of difficulty Thus the broken line alone might be introduced first, then a single 'carry' or 'borrow'; thereafter, in subtraction, a zero in the bottom row and lastly a zero in the upper row.

The Subtraction Process involving carrying should not be attempted in the Infant Division.

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